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Favorite Quotes

By Karlis Podnieks

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Jean Perrin, 1913. Explain the complicated visible by some simple invisible...

In the words of the physicist Jean Perrin, the heart of the problem is always "to explain the complicated visible by some simple invisible".

Quoted after:
François Jacob. Evolution and Tinkering. *Science*, 10 June 1977, Volume 196, Number 4295, p.1161-1166 (online copy). [Thanks to Alvis Brazma.]

French original:
“Deviner ainsi l’existence ou les propriétés d’objets qui sont au delà de notre connaissance, *expliquer du visible compliqué par de l’invisible simple*, voilà la forme d’intelligence intuitive à laquelle, grâce à des hommes tels que Dalton ou Boltzmann, nous devons l’Atomistique, ...”

See p. III of:
J. Perrin. *Les Atomes*, Librairie Félix Alcan, 1913 (online copy)

Wolfgang Stegmüller, 1969, Metaphysik – Wissenschaft – Skepsis

„Der auf sein Spezialgebiet konzentrierte wissenschaftliche Fachmann (Mathematiker, Historiker, Naturwissenschaftler) hört nicht gern, *daß fundamentale Voraussetzungen seiner Denktätigkeit metaphysischer Natur sind*; der Metaphysiker hört nicht gern, *daß seine geistige Tätigkeit auf einer vorrationalen Urentscheidung beruht*; Philosophen aller Varianten, außer Skeptikern, hören nicht gern, *daß die ernst zu nehmenden Arten der Skepsis unwiderleglich sind*; schließlich nehmen die Skeptiker aller Schattierungen nicht gern zur Kenntnis, *daß sie ihren Standpunkt nicht beweisen können*. Eine solche komplexe Feststellung provoziert geradezu das empörte Aufbehren.”

Full text:

W. Stegmüller. *Metaphysik, Skepsis, Wissenschaft*. Zweite, verbesserte Auflage. Berlin, Heidelberg, New York: Springer, 1969, pp. 1-2.

Jeffrey A. Barrett, 2001. Are our current theories true or false?

“Since we do not know the ways in which our current theories will fail and how we will address these failures, one cannot say the sense in which our current theories are true or false.”

Full text:

J. A. Barrett. Toward a Pragmatic Account of Scientific Knowledge, 7 October 2001, 43 pp. (PhilSci Archive, 498)

Nancy Cartwright, 1999. The Dappled World.

“... we live in a dappled world, a world rich in different things, with different natures, behaving in different ways. The laws that describe this world are a patchwork, not a pyramid.”

See p. 1 of: **N. Cartwright**. The Dappled World: A Study of the Boundaries of Science, Cambridge University Press, 1999, 260 pp.
John Dewey (1938) and **Charles S. Peirce** (1878). About truth. "The best definition of *truth* from the logical standpoint which is known to me is that by Peirce:" (see below) Full text: **J. Dewey.** Logic: The Theory of Inquiry. New York: Holt, 1938, p. 58 (quoted after: **D. Davidson.** Truth and Predication, Harvard University Press, 2005, p. 8) "The opinion which is fated to be ultimately agreed to by all who investigate, is what we mean by the truth, and the object represented in this opinion is the real." Full text: **C. S. Peirce.** How to Make Our Ideas Clear. *Popular Science Monthly*, 12 (January 1878), 286-302 (online copy at www.peirce.org/)
Kenneth Craik, 1943. Numbers and existence of external world. "Chapter 5 – Hypothesis on the nature of thought From this point onwards we are advancing a hypothesis and shall take the existence of the external world and of causation for granted." ... "Perhaps the extraordinary pervasiveness of number, and the multiplicity of operations which can be performed on number without leading to inconsistency, is not a proof of the 'real existence' of numbers as such but a proof of the extreme flexibility of the neural model or calculating machine." ... "We have now to enquire how the neural mechanism, in producing numerical measurement and calculation, has managed to function in a way so much more universal and flexible than any other. Our question, to emphasize it once again, is not to ask what kind of thing a number is, but to think what kind of mechanism could represent so many physically possible or impossible, and yet self-consistent, processes as number does." Full text: **K. Craik**. The Nature of Explanation. Cambridge University Press, 1943, 136 pp.
William James, 1907. What is "absolute truth"? "The 'absolutely' true, meaning what no farther experience will ever alter, is that ideal vanishing-point towards which we imagine that all our temporary truths will some day converge. It runs on all fours with the perfectly wise man, and with the absolutely complete experience; and, if these ideals are ever realized, they will all be realized together. Meanwhile we have to live to-day by what truth we can get to-day, and be ready to-morrow to call it falsehood. Ptolemaic astronomy, euclidean space,

aristotelian logic, scholastic metaphysics, were expedient for centuries, but human experience has boiled over those limits, and we now call these things only relatively true, or true within those borders of experience. 'Absolutely' they are false; for we know that those limits were casual, and might have been transcended by past theorists just as they are by present thinkers."

Full text:
W. James. Pragmatism: A New Name for Some Old Ways of Thinking. Lecture 6. Pragmatism's Conception of Truth, 1907 (published online by Project Gutenberg).

Alexander Grothendieck, 1928-2014
Photos

"Grothendieck was convinced that if one had a sufficiently unifying vision of mathematics, if one could sufficiently penetrate its conceptual essence, then particular problems would be nothing but tests that no longer need to be solved for their own sake."

Full text:
P. Cartier. Alexander Grothendieck. A Country Known Only by Name. *Inference: International Review of Science*, 1(1), October 15, 2014.

Grothendieck's most popular quote on the Internet:

"If there is one thing in mathematics that fascinates me more than anything else (and doubtless always has), it is neither "number" nor "size", but always form. And among the thousand-and-one faces whereby form chooses to reveal itself to us, the one that fascinates me more than any other and continues to fascinate me, is the structure hidden in mathematical things."

Full text:
A. Grothendieck. Récoltes et Semailles.

Andrey Kolmogorov, 1969. Finite creatures in a finite world may be forced to invent infinity.

"... let us imagine an intelligent creature living in a world of *finite* complexity, [a world] taking only a *finite* number of physically distinct states and evolutioning in a "discrete time". ... It is possible to explain plausibly how such a creature, because of its structure, being unable to cover all the complexity of the world around it and confronted with systems of growing complexity and consisting of very large numbers of elements, will create during the process of its entirely practically and reasonably oriented activities the concept of an *infinite* sequence of natural numbers." (Sorry, my own English translation.)

The original text:
A. N. Kolmogorov. Scientific foundations of the school mathematics course. First lecture. Contemporary views on the nature of mathematics, *Matematika v Shkole*, no. 3 (1969), 12–17 (in Russian). Reprinted in 1988

as pp. 232-233 in (available online).
Timothy Gowers, 2011. Observation/Discovery vs. Creation/Invention "... the distinction between discovery and observation is not especially important: if you notice something, then that something must have been there for you to notice, just as if you discover it then it must have been there for you to discover. So let us think of observation as a mild kind of discovery rather than as a fundamentally different phenomenon. " (p. 4) "... we do not normally talk of inventing a single work of art. However, we do not discover it either: a commonly used word for what we do would be 'create'. And most people, if asked, would say that this kind of creation has more in common with invention than with discovery, just as observation has more in common with discovery than with invention." (p. 5) Full text: **T. Gowers**. Is mathematics discovered or invented? In J. C. Polkinghorne (ed.), *Meaning in Mathematics*. Oxford University Press. 3-12 (2011) **My comment**: Thus, the distinction *observation/creation* is more fundamental than the (rather psychological) one of *discovery/invention*. And, the right question to ask is not: "Is mathematics discovered or invented?", but "**Is mathematics observed or created?**"
Stephen W. Hawking, 2002. We and our models are both part of the universe. "... we are not angels, who view the universe from the outside. Instead, we and our models are both part of the universe we are describing." Full text: **S. W. Hawking**. Gödel and the end of Physics. Public lecture (2002)
Akihiro Kanamori, 2009-2013. The carriers of mathematical knowledge are *proofs*. **"**What brings us mathematical knowledge? The carriers of mathematical knowledge are *proofs*, more generally arguments and constructions, as embedded in larger contexts.1 Mathematicians and teachers of higher mathematics know this, but it should be said. Issues about competence and intuition can be raised as well as factors of knowledge involving the general dissemination of analogical or inductive reasoning or the specific conveyance of methods, approaches or ways of thinking. But in the end, *what can be directly conveyed as knowledge are proofs*." Full text: **A. Kanamori.** Mathematical Knowledge: Motley and Complexity of Proof. *Annals of the Japan Association for Philosophy of Science*, Vol. 21 (2013), pp. 21-35.
Albert Einstein, 1930. Knowledge cannot spring from experience alone.

"It seems that the human mind has first to construct forms independently, before we can find them in things. Kepler's marvelous achievement is a particularly fine example of the truth that knowledge cannot spring from experience alone, but only from the comparison of the inventions of the intellect with observed fact." English translation by Sonja Bargmann published in: **A. Einstein**. Ideas and Opinions. *Crown Publishers*, New York, 1954, pp. 262-266. German original: "Es scheint, dass die menschliche Vernunft die Formen erst selbständig konstruieren muss, ehe wir sie in den Dingen nachweisen können. Aus Keplers wunderbarem Lebenswerk erkennen wir besonders schön, dass aus bloßer Empirie allein die Erkenntnis nicht erblühen kann, sondern nur aus dem Vergleich von Erdachtem mit dem Beobachteten." Full text: Albert Einstein über Kepler. *Frankfurter Zeitung*, 9. November 1930, see also online copy published by Dr. Böttiger-Verlag-GmbH.) See also Einstein's manuscript of this paper in Einstein Archives Online.
Justus von Liebig, 1865. Knowledge cannot spring from experience alone. "Die Erfindung der Elektrisirmaschine, des Elektrophors, der Leydener Flasche, der Volta'schen Säule, die drei Kepler'schen Gesetze sind durch Combinationen der Einbildungskraft erworben worden; ebenso verhält es sich mit den Verfahrungsweisen zur Gewinnung der Metalle, welche, wie die des Eisens aus den Eisensteinen, des Silbers aus den Bleierzen, des Kupfers aus den Kupfererzen etc., zu den verwickeltsten Processen gehören. Die Ueberführung des Eisens in Stahl, des Kupfers in Messing, die Verwandlung der Haut in Leder, des Fettes in Seife, die des Kochsalzes in Soda **und tausend ähnliche wichtige Erfindungen sind von Menschen gemacht worden, welche keine oder eine ganz falsche Vorstellung von der eigentlichen Natur der Dinge oder den Vorgängen hatten, an die sich ihre Ideencombination knüpfte.** [p. 6, marked bold by me, K.P.] ... **"Oft ist die Idee, von der sie ausgingen, ganz falsch, und es wird die richtige erst in der Untersuchung erweckt. Daher denn die Meinung mancher der grössten Forscher, dass die Arbeit alles mache, und dass jede Theorie zu Entdeckungen führe, vorausgesetzt, dass sie zur Arbeit antreibt."** [p. 8, marked bold by me, K.P.] Full text: **J. von Liebig.** Induktion und Deduktion. Akademische Rede, München 1865 (available online, google for *von liebig induktion*).
Karlis Podnieks. February 2010. Human minds... "Everything that is going on in human minds can be best understood as modeling."

Exposition of the idea: **K. Podnieks**. **Towards Model-Based Model of Cognition**.The Reasoner, Vol. 3, N 6, June 2009, pp. 5-6.
Philip W. Anderson, 1972. Reductionism does not imply "constructionism". "... the reductionist hypothesis does not by any means imply a "constructionist" one: The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe." ... "The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. ..., at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other." Full text: **P. W. Anderson**. More Is Different. *Science*, *New Series*, Vol. 177, No. 4047. (Aug. 4, 1972), pp. 393-396.
Philip W. Anderson, 1994. At the frontier of complexity, the watchword is not reductionism but emergence. "But another large fraction [of physicists - K.P.] are engaged in an entirely different type of fundamental research: research into phenomena that are too complex to be analyzed straightforwardly by simple application of the fundamental laws. These physicists are working at another frontier between the mysterious and the understood: the frontier of complexity. At this frontier, the watchword is not reductionism but emergence. Emergent complex phenomena are by no means in violation of the microscopic laws, but they do not appear as logically consequent on these laws." ... "(ii) One may make a digital computer using electrical relays, vacuum tubes, transistors, or neurons; the latter are capable of behaviors more complex than simple computation but are certainly capable of that; we do not know whether the other examples are capable of "mental" phenomena or not. But the rules governing computation do not vary depending on the physical substrate in which they are expressed; hence, they are logically independent of the physical laws governing that substrate." Full text: **P. W. Anderson**. Physics: The opening to complexity. *Proc. Natl. Acad. Sci. USA*, Vol. 92, pp. 6653-6654, July 1995.
Bertrand Russell, 1914. Postulates in metaphysics... "The oneness of the world is an almost undiscussed postulate of most metaphysics. ... Yet I believe that it embodies a failure to effect thoroughly the "Copernican revolution," and that the apparent oneness of the world is merely the oneness of what is seen by a single spectator or apprehended by a single mind."

Full text: **Betrand Russell**. On Scientific Method in Philosophy. *Herbert Spencer lecture at Oxford in 1914,* pp. 97-124 in: **Betrand Russell**. Mysticism and Logic: and Other Essays, *New York: Longmans, Green and Co.*, 1918 (available online). I would say: moreover, **any metaphysics is a theory based on its own postulates**, no more than that! Otherwise, we couldn't discuss several different metaphysics simultaneously.
Haskell B. Curry, 1939. One postulates the existence of an external world... "On what grounds do we infer the reality say of the table on which I am writing? I understand that one can consistently maintain the view, called solipsism, that physical objects have no reality; i.e., that the sole reality is my sensations. In fact, one does not prove the existence of an external world, one postulates it." See p. 7 of **Haskell B. Curry**. Outlines of a Formalist Philosophy of Mathematics. *North-Holland,* 1951, 80 pp.
E Brian Davies, 2007. Let Platonism die. "The beliefs of most Platonists are based on gut instincts – strong convictions reinforced by years of immersion in their subject. ... These studies [scientific investigations of mental processes – K.P.] are proceeding systematically and are beginning to provide a genuine understanding of the basis of our mathematical abilities. They owe nothing to Platonism, whose main function is to contribute a feeling of security in those who are believers. Its other function has been to provide employment for hundreds of philosophers vainly trying to reconcile it with everything we know about the world. It is about time that we recognised that mathematics is not different in type from all our other, equally remarkable, mental skills and ditched the last remnant of this ancient religion." Full text: **E. B. Davies**. Let Platonism die. *NEWSLETTER OF THE EUROPEAN MATHEMATICAL SOCIETY*, Issue 64, June 2007, pp. 24-25.
E Brian Davies, July 2001, large finite numbers only exist in a metaphysical sense "... sufficiently large finite numbers only exist in a metaphysical sense: they play no role in science and our only access to them depends upon accepting the rules of Peano arithmetic. ... ... no material object can be said to contain a precise number of atoms if that number is greater than 10^{30}. One may of course consider an *ideal* silver cube containing exactly 10^{10} atoms along each edge and therefore 10^{30} atoms altogether, but this cube is then a mental construction and not something in the material world. ...

For even bigger numbers the situation shifts again. The number of massive elementary particles in the universe is beleived to be less than $M=10^{100}$. ... If one regards all sets of particles as candidates for material entities then 2^N is an upper bound for the number of different material entities. It is a matter of fact that physicists do not make use of numbers vastly bigger than this, and it is difficult to argue that they have *any* empirical status.
...
We consider the real number field of Dedekind and Weierstrass to be metaphysical because its adds features to the physical continuum which have no empirical justification. ... Mathematicians have produced these idealized versions of the empirical continuum because of their need to have sharp formalisms if they are to prove theorems, and because the idealized systems are easier to grasp intuitively."
Full text:
E Brian Davies. Empiricism in Arithmetic and Analysis, *Philosophia Mathematica* (3) 11 (2003) 53-66.

A similar point of view is expressed in:

"Real" Analysis is a Degenerate Case of Discrete Analysis by **Doron Zeilberger** (appeared in "New Progress in Difference Equations" (Proc. ICDEA 2001), Taylor and Francis, London, 2004)

The idea can be traced to Paul Bernays in 1934, for a more detailed history, see Section 1.1 of

K. Podnieks. What is Mathematics: Gödel's Theorem and Around, 1997-2015.

Carl Johannes Thomae, 1898, the true founder of the formalist philosophy of mathematics?

"The formal conception of numbers requires of itself more modest limitations than does the logical conception. It does not ask, what are and what shall the numbers be, but it asks, what does one require of numbers in arithmetic. For the formal conception, arithmetic is a game with signs which one may call empty; by this one wants to say that (in the game of calculation) they have no other content than that which has been attributed to them concerning their behaviour with respect to certain rules of combination (rules of the game). Similarly a chess player uses his pieces, he attributes to them certain properties which condition their behaviour in the game, and the pieces themselves are only external signs for this behaviour. To be sure, there is an important difference between the game of chess and arithmetic. The rules of chess are arbitrary; the system of rules for arithmetic is such that by means of simple axioms the numbers can be related to intuitive manifolds, so that they are of essential service in the knowledge of nature. - The formal standpoint relieves us of all metaphysical difficulties, that is the benefit it offers to us."

Quoted after:
Carl Johannes Thomae in the MacTutor History of Mathematics archive at the University of St Andrews.
English translation may be due to:
Moritz Epple. Chapter 10 of: A history of analysis. Hans Niels Jahnke

(ed.). *American Mathematical Society*, Providence, 2003.
The original text:
Carl Johannes Thomae (1840-1921). Elementare Theorie der analytischen Funktionen einer komplexen Veränderlichen. *Zweite erweiterte und umgearbeitete Auflage*. Halle, 1898.

Paul Dirac on interpretations, before 1984

"... Dirac was never interested in interpretations [of quantum theory – K.P.]. It seemed to him to be a pointless preoccupation that led to no new equations."
Full text: p. 277 of
Manjit Kumar. Quantum: Einstein, Bohr and the Great Debate About the Nature of Reality. *Icon Books Ltd*, 2008, 480 pp.
"In these cases, as in the case of quantum mechanics, a very strictly empiricist position could have circumvented the problem [of interpretation – K.P.] altogether, by reducing the content of the theory to a list of predicted numbers. But perhaps science can offer us more than such a list; and certainly **science needs more** than such a list **to find its ways** [marked bold by me – K.P.]."
Full text:
Federico Laudisa, **Carlo Rovelli**. Relational Quantum Mechanics. *Stanford Encyclopedia of Philosophy*, 2008.

Thus, interpretations may be useful, but only when they "lead to new equations".

Richard Feynman, between 1918 and 1988. Electron is a theory...

"**The electron is a theory that we use; it is so useful in understanding the way nature works that we can almost call it real.** I wanted to make the idea of a theory clear by analogy. In the case of the brick, my next question was going to be, "What about the inside of the brick?" - and I would then point out that no one has ever seen the inside of a brick. Every time you break the brick, you only see the surface. **That the brick has an inside is a simple theory which helps us understand things better.** The theory of electrons is analogous." (Fragments marked bold by me – K. P.)

Full text – in Chapter 9 of:
Surely You're Joking, Mr. Feynman!: Adventures of a Curious Character, with contributions by Ralph Leighton, *W. W. Norton & Co*, 1985.

Werner Heisenberg, March 1927. We cannot know the present in all detail.

"But what is wrong in the sharp formulation of the law of causality, "When we know the present precisely, we can predict the future," is not the conclusion but the assumption. Even in principle we cannot know the present in all detail."

German original: "Aber an der scharfen Formulierung des Kausalgesetzes: "Wenn wir die Gegenwart genau kennen, koennen wir die Zukunft berechnen", ist nicht der Nachsatz, sondern die Voraussetzung falsch. Wir

koennen die Gegenwart in allen Bestimmungstuecken prinzipiell n i c h t kennenlernen." See p. 198 of: **W. Heisenberg**. Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik, *Zeitschrift für Physik*, Volume 43, pp. 172-198 (1927), online copy available. *English translation*: Quantum Theory and Measurement; Wheeler, J. A.; Zurek, W. H., Eds.; *Princeton University Press*: Princeton, NJ, 1983; pp. 62-84.
Alan Musgrave, 1989, Deductive logic is the only logic that we have or need! "... reasoners seldom, if ever, state all of the premises they are assuming. We usually, perhaps always, have to reconstruct the arguments being employed. Deductivism is the view that deductive logic is the only logic that we have or need. Deductivists can always reconstruct what look like non-deductive or inductive arguments as deductive arguments with missing premises of one kind or another. ... it conduces to clarity, if we do treat them so. ...if accepted, it would enable us to make good Popper's claim that induction is a myth." See p. 319 of: **A. Musgrave**. Essays on Realism and Rationalism. *Rodopi*, Amsterdam-Atlanta, 1999, pp. xiii + 373.
Stephanie Ruphy, 2008, *The Millennium Run* – balancing ambition and outcome... "Acknowledging path dependency immediately puts to the fore the contingency of a simulation such as the Millennium Run. Had the cosmologists chosen different options at some stages in the model-building process, they would have come up with a different picture of the evolution of cosmic matter. And the point is that those alternative pictures would be equally plausible in the sense that they would also be consistent both with the observations at hand and with our current theoretical knowledge." **S. Ruphy**. Limits to Modeling: Balancing Ambition and Outcome in Astrophysics and Cosmology. In: *Simulation & Gaming*, first published on June 2008 as doi: 10.1177/ 1046878108319640, Sage Publications.
Jeff Rothenberg, 1989, We are, ... modelers... Modeling underlies our ability to think and imagine, to use signs and language, to communicate, to generalize from experience, to deal with the unexpected, and to make sense out of the raw bombardment of our sensations. It allows us to see patterns, to appreciate, predict, and manipulate processes and things, and to express meaning and purpose. In short, it is one of the most essential activities of the human mind. It is the foundation of what we call intelligent behavior and is a large part of what makes us human. We are, in a word, modelers: creatures that build and use models routinely, habitually−−sometimes even compulsively−−to face, understand, and interact with reality.

Full text: **J. Rothenberg**. THE NATURE OF MODELING. *AI, Simulation & Modeling*, John Wiley & Sons, 1989, pp. 75−92.

Nancy Cartwright, 1983, How the Laws of Physics Lie

“My basic view is that fundamental equations do not govern objects in reality; they only govern objects in models.”

See p. 129 of:
N. Cartwright. How the Laws of Physics Lie. *Oxford University Press*, 1983, 232 pp.

[Attributed to] **Niels Bohr**, before 1963.

"There is no quantum world. There is only an abstract quantum physical description. It is wrong to think the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature."

Quoted after:
Aage Petersen (1963: The Philosophy of Niels Bohr. *Bulletin of the Atomic Scientists*, XIX(7): 8-14).

Barry Mazur, June 2008, Experience of platonic realms

"When I’m working I sometimes have the sense – possibly the illusion – of gazing on the bare platonic beauty of structure or of mathematical objects, and at other times I’m a happy Kantian, marvelling at the generative power of the intuitions for setting what an Aristotelian might call the *formal conditions of an object*. And sometimes I seem to straddle these camps (and this represents no contradiction to me). I feel that the intensity of this experience, the vertiginous imaginings, the leaps of intuition, the breathlessness that results from “seeing” but where the sights are of entities abiding in some realm of ideas, and the passion of it all, is what makes mathematics so supremely important for me. Of course, the realm might be illusion. But the experience?"

Full text:
B. Mazur. Mathematical Platonism and its Opposites. *NEWSLETTER OF THE EUROPEAN MATHEMATICAL SOCIETY*, Issue 68, June 2008, pp. 17-18 (online copy).

Reuben Hersh, June 2008, Mathematics before Big Bang?

"I once took a vote in a talk at New Mexico State University in Las Cruces. The question was, “Was the spectral theorem on self-adjoint operators in Hilbert space true before the Big Bang, before there was a universe?” The vote was yes, by a margin of 75 to 25. But there were no self-adjoint operators, no Hilbert space, before the twentieth century!"

Full text:
R. Hersh. On Platonism. *NEWSLETTER OF THE EUROPEAN*

MATHEMATICAL SOCIETY, Issue 68, June 2008, pp. 19-21 (online copy).
Deena Skolnick Weisberg on mathematical platonism, about 2008. "To use a phrase suggested by Deena Weisberg, a view of model systems as imagined concrete things which many scientists can simultaneously investigate is the "folk ontology" of model-based science, the ontology that is implicit in the practitioners' routine behaviors. ... an implicitly platonist outlook is a feature of successful mathematical practice – in Weisberg's terms again, ... platonism is the folk ontology of research mathematics." Quoted after: **Peter Godfrey Smith**. Models and Fictions in Science. *Philosophical Studies* 143 (2009): pp. 101-116. My comment: "Implicitly platonist outlook is a feature of **successful** mathematical practice" - I'm promoting this idea since 1988 when I first presented it at the Heyting'88 Summer School & Conference on Mathematical Logic (view copy of Abstract, a detailed exposition).
Doron Zeilberger, Computer assisted mathematics, January 2008 "... it is very possible that if Andrew Wiles' programming skills would have been as good as his proving skills, he would have already proved the Riemann Hypothesis." Full text: http://www.math.rutgers.edu/~zeilberg/Opinion94.html.
Karlis Podnieks. Infinity, January 2006 "Couldn't the invention of the axiom of infinity simply be an act of fantasy?" Full text: [FOM] Infinity and the "Noble Lie".
Freek Wiedijk. The Future of Formal Mathematics, November 2008 In a few decades it will no longer take one week to formalize a page from an undergraduate textbook. Then that time will have dropped to a few hours. Also then the formalization will be quite close to what one finds in such a textbook. When this happens we will see a quantum leap, and suddenly all mathematicians will start using formalization for their proofs. When the part of refereeing a mathematical article that consists of checking its correctness takes more time than formalizing the contents of the paper

would take, referees will insist on getting a formalized version before they want to look at a paper.

However, having mathematics become utterly reliable might not be the primary reason that eventually
formal mathematics will be used by most mathematicians. Formalization of mathematics can be a very rewarding activity in its own right. It combines the pleasure of computer programming (craftsmanship, and the computer doing things for you), with that of mathematics (pure mind, and absolute certainty.) People who do not like programming or who do not like mathematics probably will not like formalization. However, for people who like both, formalization is the best thing there is. (P. 1414)

Full text:
Freek Wiedijk. Formal Proof - Getting Started. Notices of the AMS, 2008, Vol. 55, N 11, pp. 1408-1414 (available online).

Richard Feynman. The Nobel Prize in Physics, 1965.

"The fact that electrodynamics can be written in so many ways ..., was something I knew, but I have never understood. It always seems odd to me that the fundamental laws of physics, when discovered, can appear in so many different forms that are not apparently identical at first, but, with a little mathematical fiddling you can show the relationship. ... I don't know why this is - it remains a mystery, but it was something I learned from experience. There is always another way to say the same thing that doesn't look at all like the way you said it before. I don't know what the reason for this is. I think it is somehow a representation of the simplicity of nature. ... I don't know what it means, that nature chooses these curious forms, but maybe that is a way of defining simplicity. Perhaps a thing is simple if you can describe it fully in several different ways without immediately knowing that you are describing the same thing."

Full text:
R. Feynman. The Development of the Space-Time View of Quantum Electrodynamics. Nobel Lecture, December 11, 1965.

Daniel C. Dennett, 2008

"A point I have often made is that computer science keeps cognitive science honest. If it weren't for the practical possibility of constructing and demonstrating simplified working models of cognitive processes, we'd still be at the hand-waving stage. ... most of the good work in computer science (and related fields such as robotics) enlarges our appreciation for just how remarkable our brains are.

Full text:
Interview for "Philosophy of Computing and Information - Five Questions", edited by Luciano Floridi, forthcoming.

Hilary Putnam: Philosophy of Mathematics: **Why Nothing Works?**

Question asked 1979, the answer of 1997 follows (as put by Wikipedia):

"Under the influence of Ludwig Wittgenstein, he [Putnam - K.P.] adopted a pluralist view of philosophy itself and came to view most philosophical problems as nothing more than conceptual or linguistic confusions created by philosophers by using ordinary language out of its original context."

Full text: Hilary Putnam by Wikipedia.
See also:
Putnam, H. "Philosophy of Mathematics: Why Nothing Works" in *Words and Life*, Harvard University Press, 1994, pp. 499-512 (written in 1979).
Putnam, H. "A Half Century of Philosophy: Viewed from Within". *Daedalus*, 1997, (12).

Donald E. Knuth, 20 May 1995

"Science is what we understand well enough to explain to a computer. Art is everything else we do."

Full text:
Foreword of the book: **A=B**, by Marko Petkovsek, Herbert Wilf and Doron Zeilberger, A K Peters Ltd, 1996, 224 pp. (online copy).

David Hestenes, 1992, **[**models are **invented,** it is their fitness to data that is **discovered!]**:

"Kepler employed the Copernican reference system in his own analysis and showed that Tycho's more accurate data could not be fitted to the Copernican model. ... Since no one before had ever considered any kinematical alternative to uniform circular motion, Kepler had to *invent* his own to fit the data. His brilliant result is formulated as a system of functional relations called Kepler's laws. Many physicists would insist that Kepler's laws were discovered rather than invented. On the contrary, what Kepler *discovered* was that these laws fit the data. He had considered and discarded many alternatives. It would be better to speak of *Kepler's model* (rather than laws) and say that the model has been *validated* to the precision in Tycho's observations. We now know that many alternative models could be invented to fit the same data, but Kepler's is the simplest of all models in this class. We also know that Kepler's could not fit the more accurate data collected with telescopes rather than the naked eye, because the elliptical planetary orbits are perturbed by gravitational forces from other planets, ... That fact could never be discovered by Kepler's method; the more powerful method of Newton was needed. It is no small irony that Newton's law of gravitation would undoubtedly have been more difficult to discover if Kepler's model had been quickly invalidated by more accurate data. Here we have the possibility that scientific progress might be impeded by greater experimental precision."

Full text:
D. Hestenes. Modeling Games in the Newtonian World. *American*

Journal of Physics, 1992, Volume 60, Issue 8, pp. 732-748.

L E J Brouwer, 19XX

"Mathematics is nothing more, nothing less, than the exact part of our thinking."

See Quotations by L E J Brouwer in The MacTutor History of Mathematics archive.

Patrick Suppes, 1978

"... I do want to convey the basic philosophical point that I continue to find the real puzzle of quantum mechanics. Not the move away from classical determinism, but the ways in which the standard versions seem to lie outside the almost universal methodology of modern probability theory and mathematical statistics. For me it is in this arena that the real puzzles of quantum mechanics are to be found. I am philosophically willing to violate classical physical principles without too many qualms, but when it comes to moving away from the broad conceptual and formal framework of modern probability theory I am at once uneasy. My historical view of the situation is that if probability theory had been developed to anything like its current sophisticated state at the time the basic work on quantum mechanics was done in the twenties, then a very different sort of theory would have been formulated."

Full text:
P. Suppes. Intellectual Autobiography, 1978, p.5.

"For example, in quantum chemistry there is, with present intellectual and computing resources, no hope of making a direct attack on the behavior of complex molecules by beginning with the first principles of quantum theory. A problem as easy to formulate as that of deriving from first principles the boiling point of water under normal atmospheric pressure is simply beyond solution at the present time and is recognized as such."

Full text: p.15.

"... On the other hand, I strongly believe that a reduction of psychology to the biological or physical sciences will not occur and is not intellectually feasible. I am not happy with leaving the statement of my views at this level of generality, and I consider it an intellectual responsibility of methodological behaviorists like myself to reach for a deeper and more formal statement of this antireductionist position. What are needed are theorems based on currently reasonable assumptions showing that such a reduction cannot be made. I think of such theorems as being formulated in the spirit in which theorems are stated in quantum mechanics about the impossibility of deterministic hidden variable theories."

Full text: p.16.

Paul Bernays, 1958

"As Bernays remarks, syntax is a branch of number theory and semantics the one of set theory."

See p. 470 of
Hao Wang. EIGHTY YEARS OF FOUNDATIONAL STUDIES. *Dialectica*, Vol. 12, Issue 3-4, pp. 466-497, December 1958.

David Hestenes, 2006

"To the grand philosophical question: "*What is a man?*" Aristotle answered: "*Man is a rational animal.*" Modeling Theory offers a new answer: "*Man is a modeling animal!*" HOMO MODELUS!"

D.Hestenes. Notes on Modeling Theory, *Proceedings of the 2006 GIREP conference: Modelling in Physics and Physics Education*.

From: **Vladimir Sazonov**
Sent: *Tue Jul 31 15:47:24 EDT 2007*
Subject: Re: [FOM] Sazonov on intuitive and formal mathematics

"When trying to formalize some new imaginary world (say, of infinite objects - sets) we should realize that it is only imaginary one and nothing is true or false there in the same sense as in the ordinary physical world or for not so big finite objects, and no logical laws hold there just because they are "objectively" true - "do not hold" in this sense. Thus, when we DECIDE to impose the ordinary (or any other preferable) logical laws onto this world, it is not because they are true there. It is because this is OUR decision, assuming it is sufficiently coherent with our imagination and sufficiently robust. (The coherence is typically incomplete; also various surprises - counterexamples - are possible which would rather "correct" our intuition. And we so much respect these formalisms that we usually are quite happy with these corrections.) We create our own worlds and "play" there by the laws of some logic we choose, let Aristotelian."

Full text at http://www.cs.nyu.edu/pipermail/fom/2007-July/011784.html

John McCarthy, February 29, 1996

"It turns out that many philosophical problems take new forms when thought about in terms of how to design a robot."

Full text:
J. McCarthy. What has AI in Common with Philosophy?, 1996

Carl Friedrich Gauss to Franz Adolph Taurinus, Goettingen, November 8, 1824

"... But it seems to me that in spite of the word-mastery of the metaphysicians, we know really too little, or even nothing at all, about the true nature of space to be able to confuse something that seems unnatural with *absolutely impossible*. If non-Euclidean geometry is the real one and the constant is incomparable to the magnitudes that we encounter on earth or in the heavens then it can be determined aposteriori. I have therefore occasionally for fun expressed the wish that Euclidean geometry not be the real one, for then we would have a priori an absolute measure."

Full text: Gauss And Non-Euclidean Geometry by Stanley N. Burris (see also the *German original* at the *Göttinger Digitalisierungs-Zentrum*).

My comment:
Try replacing "Euclidean geometry" by "large cardinal axioms", and "non-Euclidean geometry" - by Petr Vopenka's Alternative Set Theory. (Pictures, Russian translation)

Aleksandr Danilovic Aleksandrov, about mathematics as a kind of technology, 1970.

"Mathematics is creating its apparatus, and speaking about its truth or falsity is senseless: the apparatus is either working, or not working, and if working, it is working either productively, or not. A similar nonsense would be asking: "Is this screwdriver true or false?"; the screwdriver simply *exists*, and one can only ask sensibly, how is it working, and where could it be applied." (continue here, in Russian, sorry, my own English translation).

Full text:
A. D. Aleksandrov. Mathematics and dialectics. Siberian Mathematical Journal, 1970, Vol.11, pp.185-197.

Hilary Putnam, about "theory-dependence of meaning and truth", December 29, 1977.

"It may well be the case that the idea that statements have their truth values *independent* of embedding theory is so deeply built into our ways of talking that there is simply no "ordinary language" word or short phrase which refers to the theory-dependence of meaning and truth. Perhaps this is why Poincare was driven to exclaim "Convention, yes! Arbitrary, no!" when he was trying to express a similar idea in another context." (p. 471)

"... The language, on the perspective we talked ourselves into, has a full program of use; but it still lacks *interpretation*.

This is the fatal step. To adopt a theory of meaning according to which a language whose whole use is specified still lacks something - viz. its "interpretation" - is to accept problem which *can* only have crazy solutions. To speak as if *this* were my problem, "I know how to use my language, but, now, how shall I single out an interpretation?" is to speak nonsense. Either the use *already* fixes the "interpretation" or *nothing* can." (pp.481-482)

Full text:
H. Putnam. Models And Reality. Journal of Symbolic Logic, September 1980, Vol.45, N3, pp. 464-482.

Jan Mycielski, at the Russell's Paradox Centennial Conference, Munich, 2001

"... In this state of affairs the existence of Platonists in this day and age is puzzling to us as it was puzzling to Tarski ... and presumably to Russell. ... Of course there were very outstanding Platonists, and among them Frege, Zermelo and Gödel. ..."

Full text:
J. Mycielski. Russell's Paradox and Hilbert's (much forgotten) View of Set Theory. *One Hundred Years of Russell's Paradox: Mathematics, Logic, and Philosophy*. Berlin, New York: Walter de Gruyter, 2004, pp. 533-547 (online copy: PDF).

Alfred Tarski, at the "Tarski Symposium", University of Berkeley, 1971:

"People have asked me 'How can you, a nominalist, do work in set theory and logic, which are theories about things you do not believe in?' . . . I believe there is value even in fairy tales and the study of fairy tales."

Quoted after:
Charles Chihara letter to Anita Burdman Feferman, June 1994,
see p.52 of :
Alfred Tarski: Life and Logic. *By Anita Burdman Feferman and Solomon Feferman, Cambridge University Press, Cambridge, UK, 2004, 432 pp.*

From: **Timothy Y. Chow**
Sent: *Sun Oct 22 16:15:25 EDT 2006*
Subject: Re: [FOM] First-order arithmetical truth

"Any skepticism about the naive (resp. formal) concept of the integers carries over directly into skepticism about the naive (resp. formal) concept of a formal system. Anyone who thinks that formal systems are crystal clear while integers are vague and suspect---and who tries to argue that the existence of nonstandard models lends support to that idea---is simply suffering from a blindspot that prevents him from seeing that his skeptical arguments apply equally to formal systems. "

Full text at http://www.cs.nyu.edu/pipermail/fom/2006-October/011009.html

From: **Vladimir Sazonov**

Sent: *Tue Oct 24 18:30:13 EDT 2006*
Subject: Re: [FOM] First-order arithmetical truth

"I definitely do not suffer from such a blindspot. Moreover, I make clear distinction between naive and abstract (meta)mathematical concepts discussed...

You are right. Abstract (meta)mathematical formal systems can have nonstandard formulas and derivations. Exactly the same as for abstract mathematical numbers.
...

In place of that person I would replace here "standard formal system" by "NAIVE, CONCRETE formal system" because it is continued with "on a sheet of paper". This is not about an abstract (meta)mathematical concept. This is from real human activity of writing symbols, symbolically presented rules and practical ability to follow these rules. No theory explaining this activity is needed. People just are able to do this in practice. That is why this activity is both NAIVE and CONCRETE. "

Full text at http://www.cs.nyu.edu/pipermail/fom/2006-October/011018.html

My comment:
Indeed, as Henry Poincare noticed in his book "Science et methode" (Paris, 1908, see Volume II, Chapters III and IV): (in modern terms) the idea of a "formal theory of natural numbers" is based on *petitio principii*. The abstract notion of formal syntax includes the same induction principle that is formalized in this "formal theory of natural numbers". Two possible exits from this situation are: a) regard natural numbers as a consistent notion that is independent of any definitions (platonism, a kind of mysticism), b) conclude that natural numbers represent an inconsistent notion (formalism, accepting its own consequences).

Roger Penrose about Gödel's Incompleteness Theorem, 2005:

"... he [Gödel] demonstrated that, if we are prepared to accept that the rules of some such formal system F are to be trusted as giving us only mathematically correct conclusions, then we must also accept, as correct, a certain clear-cut mathematical statement G(F), while concluding that G(F) is not provable by the methods of F alone. Thus, Gödel shows us how to transcend any F that we are prepared to trust."

Full text:
R. Penrose. The Road to Reality: A Complete Guide to the Laws of the Universe. Knopf, 2005. Thanks to *Dainis Zeps*.

My comment:
We have a simple misunderstanding here. Let's de-mystify the situation. To "accept G(F) as correct", one doesn't need trusting F "as giving us only mathematically correct conclusions". To prove G(F) "as true" we need a much weaker assumption - that F is *syntactically* consistent, i.e.

that it does not allow deriving of contradictions. This assumption can be formalized as a certain arithmetical statement Con(F). After this, we can prove the statement Con(F)->G(F) *in first order arithmetic*. Thus, here, the only heroic act is postulation of Con(T) "as true". Is free of charge postulation of Con(F) an honest way "how to transcend any F that we are prepared to trust"?

Rebecca Goldstein about Kurt Gödel, 2005:

"I'm saddened by the sense of his isolation, by how profound it must have been. It's chilling to consider that he felt the world to be so hostile that he believed his food was being poisoned and so stopped eating and so starved to death. I've spent a long time imagining what that must have felt like for such a man. And I contrast that dark and cold place in which he lived many long years and in which he ended his life with the sense of bright wonderment that I experienced that summer before graduate school, when I first understood Gödel's masterpiece of reason. He gave that experience to countless people, and we're grateful."

Full text:
Edge: GÖDEL AND THE NATURE OF MATHEMATICAL TRUTH
[6.8.05] A Talk with Rebecca Goldstein.

Michael Aschbacher about probably the most complicated mathematical proof ever, 2004-2005

"To my knowledge the main theorem of [AS] closes the last gap in the original proof, so (for the moment) the Classification Theorem can be regarded as a theorem. On the other hand, I hope I have convinced you that it is important to complete the program by carefully writing out a more reliable proof in order to minimize the chance of other gaps being discovered in the future."

Full text:
M. Aschbacher. The Status of the Classification of the Finite Simple Groups. *Notices of the AMS*, August 2004, vol. 51, N 7, pp. 736-740 (online copy)

(For the formulation of the Classification Theorem, see Classification of finite simple groups by Wikipedia, the free encyclopedia.)

"Conventional wisdom says the ideal proof should be short, simple, and elegant. However there are now examples of very long, complicated proofs, and as mathematics continues to mature, more examples are likely to appear. Such proofs raise various issues. For example it is impossible to write out a very long and complicated argument without error, so is such a ‘proof’ really a proof? What conditions make complex proofs necessary, possible, and of interest? Is the mathematics involved in dealing with information rich problems qualitatively different from more traditional mathematics?"

Full text:
M. Aschbacher. Highly complex proofs and implications of such proofs. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, October 15, 2005, vol. 363, N 1835, pp. 2401-1406 (online copy).

Alan M. Turing about "the real moral of the Gödel result" (February 20, 1947)

"As Penrose himself notes, this seems to be what Turing thought was the real moral of the Gödel result. Turing is worth quoting at length:"

"It might be argued that there is a fundamental contradiction in the idea of a machine with intelligence. It is certainly true that 'acting like a machine', has come to be synonymous with lack of adaptability... It has for instance been shown that with certain logical systems there can be no machine which will distinguish provable formulae of the system from unprovable... Thus if a machine is made for this purpose it must in some cases fail to give an answer. On the other hand, if a mathematician is confronted with such a problem he would search around and find new methods of proof, so that he ought to be able to reach a decision about any given formula. Against it I would say that fair play must be given to the machine. Instead of it sometimes giving no answer we could arrange it so that it gives occasional wrong answers. But the human mathematician would likewise make blunders when trying out new techniques. It is easy for us to regard these blunders as not counting and give him another chance, but the machine would probably be allowed no mercy. In other words then, if a machine is expected to be infallible, it cannot also be intelligent. There are several mathematical theorems which say almost exactly that. But these theorems say nothing about how much intelligence may be displayed if a machine makes no pretense at infallibility."

Quoted after:

Rick Grush, Patricia S. Churchland. GAPS IN PENROSE'S TOILINGS. *Journal of Consciousness Studies*, 1995, vol. 2, N 1, pp. 10-29 (online copy).

Full text:
A. Turing. Lecture to the London Mathematical Society on 20 February 1947. In *A.M. Turing's ACE report of 1946 and other papers* (eds. B.E. Carpenter and R.W. Doran). The Charles Babbage Institute Reprint Series for the History of Computing, Cambridge, MIT Press, 1986, vol. 10.

From: **Vladimir Sazonov**
Sent: Thursday, April 06, 2006 1:42 AM
Subject: Re: [FOM] Clarity in fom and problem solving

" I think that, like in engineering, mathematicians (or just our ancestors when they invented numbers) are really creators of mathematical concepts via formal systems, axioms, definitions, algorithms etc., and, again like in engineering, these creations are not absolutely free. However, they are *potentially* free. There is no restriction to create possibly useless/meaningless formalisms, incorrect proofs, etc. like useless (but may be amazing) engineering devices. I see mathematics in general as the engineering of formal tools (formalisms) strengthening our abstract thought, and it is this what imposes the restriction under discussion. This can be also compared with software engineering which however is devoted to mechanising the routine part of our intellectual activity."

Full text at http://www.cs.nyu.edu/pipermail/fom/2006-April/010347.html

Seth Lloyd about computational capacity of the Universe, 2002

"All physical systems register and process information. The laws of physics determine the amount of information that a physical system can register (number of bits) and the number of elementary logic operations that a system can perform (number of ops). The Universe is a physical system. The amount of information that the Universe can register and the number of elementary operations that it can have performed over its history are calculated. The Universe can have performed 10^{120} ops on 10^{90} bits (10^{120} bits including gravitational degrees of freedom)."

Full text:
S. Lloyd. Computational capacity of the universe. *Physical Review Letters*, 2002, vol. 88, issue 23, 4 pages (available online, see also 17 page extended online version).

From **Hendrik J. Boom**
Sent: Thursday, February 02, 2006 10:59 PM
Subject: [FOM] ...

"Practicing physicist seem to act as if every set of real numbers is measurable, for example. This lets them off the escalator of ascending set-theoretic axioms rather early, as they never get around to accepting the axiom of choice.

In fact, some physists seem to act as if every total function from R->R is continuous! Does this make them unwitting constructivists?"

Full text at http://www.cs.nyu.edu/pipermail/fom/2006-February/009667.html

Hilary Putnam about experimental mathematics, 1975

"In this paper, I have stressed the importance of quasi-empirical and even downright empirical methods in mathematics. ... None of this is meant to downgrade the notion of proof. Rather, Proof and Quasi-empirical inference are to be viewed as complementary. Proof has the great advantage of not increasing the risk of contradiction, where the introduction of new axioms or new objects does increase the risk of contradiction, at least until a relative interpretation of the new theory in some already accepted theory is found. For this reason, proof will continue to be the primary method of mathematical verification."

Full text:
H. Putnam. What is Mathematical Truth? *Historia Mathematica* 2 (1975): 529-543 (for reprints see Hilary Putnam Bibliography)

Edward Nelson. Mathematics and Faith. Vatican, May 23-24, 2000

"The notion of truth in mathematics is irrelevant to what mathematicians do, it is vague unless abstractly formalized, and it varies according to philosophical opinion. In short, it is formal abstraction masquerading as reality."

Full text at http://www.math.princeton.edu/~nelson/papers.html.

As a regular term, **"platonism in mathematics"** is used since the lecture delivered June 18, 1934, University of Geneva, by **Paul Bernays**:

P. Bernays. Sur le platonisme dans les mathematiques. *L'enseignement mathematique*, Vol. 34 (1935), pp. 52-69. Quoted from English translation by Charles D. Parsons at www.phil.cmu.edu/projects/bernays/Pdf/platonism.pdf.

Bernays considers mathematical platonism as a method that can be - "taking certain precautions" - applied in mathematics. Some remarkable quotes (fragments marked bold by me - K. P.):

... allow me to call it "platonism".

... The value of **platonistically inspired mathematical conceptions** is that they furnish models of abstract imagination. These stand out by their simplicity and logical strength. They form representations which extrapolate from certain regions of experience and intuition.

... This brief summary will suffice to characterize **platonism and its application to mathematics**. This application is so widespread that it is not an exaggeration to say that **platonism reigns today in mathematics**.

... Several mathematicians and philosophers interpret **the methods of platonism** in the sense of conceptual realism, postulating the existence of a world of ideal objects containing all the objects and relations of mathematics. It is this absolute platonism which has been shown untenable by the antinomies, particularly by those surrounding the Russell-Zermelo paradox.

... It is also this transcendent character which requires us to **take certain precautions in regard to each platonistic assumption.** For even when such a supposition is not at all arbitrary and presents itself naturally to the mind, it can still be that the principle from which it proceeds permits only a restricted application, outside of which one would fall into contradiction. We must be all the more careful in the face of this possibility, since the drive for simplicity leads us to make our principles as broad as possible. And the need for a restriction is often not noticed. This was the case, as we have seen, for the principle of totality, which was pressed too far by absolute platonism. Here it was only the discovery of the Russell-Zermelo paradox which showed that a restriction was necessary.

[Inspired by reading: **Jacques Bouveresse**. On the Meaning of the Word 'Platonism' in the Expression 'Mathematical platonism'. *Proceedings of the Aristotelian Society*, September 2004, Volume 105, pp. 55-79 (online French version). Thanks to William J. Greenberg.]

Max Planck (1858-1947)
Wissenschaftliche Selbstbiographie, published in 1948

"Die Wahrheit triumphiert nie, ihre Gegner sterben nur aus."
"Truth never triumphs - its opponents just die out."

Condensed version from the article Max Plack in Wikiquote, full text ibid.

John von Neumann
The Mathematician, 1947

"I think that it is a relatively good approximation to truth - which is much too complicated to allow anything but approximations - that mathematical ideas originate in empirics, although the genealogy is sometimes long and obscure. But, once they are so conceived, the subject begins to live a peculiar life of its own and is better compared to a creative one, governed by almost entirely aesthetical motivations, than to anyting else and, in particular, to an empirical science. There is, however, a further point which, I believe, need stressing.

As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired from ideas coming from 'reality', it is beset with very grave dangers. It becomes more and more purely aestheticizing, more and more purely *l'art pour l'art*. This need not be bad, if the field is surrounded by correlated subjects, which still have closer empirical connections, or if the discipline is under the influence of men with an exceptionally well-developed taste.

But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganized mass of details and complexities.

In other words, at a great distance from its empirical source, or after much 'abstract' inbreeding, a mathematical subject is in danger of degeneration. At the inception the style is usually classical; when it shows signs of becoming baroque the danger signal is up. It would be easy to give examples, to trace specific evolutions into the baroque and the very high baroque, but this, again, would be too technical.

In any event, whenever this stage is reached, the only remedy seems to me to be the rejuvenating return to the source: the reinjection of more or less directly empirical ideas. I am convinced that this was a necessary condition to conserve the freshness and the vitality of the subject, and that this will remain equally true in the future."

John von Neumann. The Mathematician, in: *The Works of the Mind*, Robert B. Heywood (ed.), University of Chicago Press, 1947, pp.180-196

Quoted after the October 22, 1995 posting by Louis A. Talman at Math Forum @ Drexel.

Antony Jay
Management and Machiavelli, 1967

"In corporation [corporate] religions as in others, the heretic must be cast out not because of the probability that he is wrong but because of the possibility that he is right." - as quoted twice by **Edsger W. Dijkstra**, February, 27, 1975 and August 11, 1982.

Doron Zeilberger, 1993
THEOREMS FOR A PRICE: Tomorrow's Semi-Rigorous Mathematical Culture

"The computer has already started doing to mathematics what the telescope and microscope did to astronomy and biology. In the future, not all mathematicians will care about absolute certainty, since there will be so many exciting new facts to discover: mathematical pulsars and quasars that will make the Mandelbrot set seem like a mere Jovian moon. We will have (both human and machine) professional *theoretical* mathematicians, who will develop conceptual paradigms to make sense out of the empirical data, and who will reap Fields medals along with (human and machine) *experimental* mathematicians. Will there still be a place for *mathematical* mathematicians?"

Full text appeared in *Notices of the AMS*, Vol. 40, N8 (October 1993), pp.978-981 (online copy available).

Roger Bishop Jones, 2005
On *how many* things there *might* be

"We find ourselves at the beginning of the 21st century facing the prospect that knowledge may no longer be the exclusive domain of human intelligence, and that questions of ontology, mathematics, science and engineering may be entertained and resolved by fabrications in silicon."

Online text in progress, February 11, 2005, at
http://www.rbjones.com/rbjpub/www/papers/p003.pdf

David Ruelle, 1999
Mathematical Platonism Reconsidered

"Admittedly, the mathematician's ideas reside in a modest amount of jelly-like substance which constitutes the mathematician's brain, ..."

"... because a mathematician's world is a world of ideas as envisioned by Plato. But what comes out of our discussion is that these ideas are very specifically human, depending on the very special organization of our brain, and in particular on its shortcomings."

"The fundamental limitations put by physical law on computing, or doing mathematics, do not appear to be very well understood at this time. ... It seems possible, however, that another crisis of foundations of mathematics may be awaiting us, and that collision with physical law could cause further damage to our Platonist conception of mathematics."

Johann Bernoulli lecture, Groningen, April 20, 1999, full text at
http://www.ihes.fr/~ruelle/PUBLICATIONS/127plato.pdf

From: **Harvey Friedman** ...
Sent: Monday, January 17, 2005 4:09 AM
Subject: [FOM] Atlanta Meeting

...
I asked the panel members whether they were interested in this line of investigation: no simple axiom settling the continuum hypothesis.
Woodin responded by saying that, overwhelmingly, he really wanted to know whether the continuum hypothesis is true or false. He is far more interested in pursuing that, as he is now, than any considerations of simplicity, which for him, was a side issue.
Martin responded by saying that the projective determinacy experience showed that one could have axioms with simple statements, but with very complicated explanations as to why they are correct.
I did not have an opportunity to respond to Martin's statement - I would have said that by the standards of axioms for set theory, projective determinacy is NOT simple. It is far more complicated than any accepted axiom for set theory.
I did ask Cohen specifically to comment on whether simplicity (of a new axiom to settle the continuum hypothesis) was important for him. Cohen responded by saying that such an axiom, for him, must be simple.
Let me end here with something concrete. In my papers in Fund. Math., and in J. Math. Logic, it is proved that all 3 quantifier sentences in set theory (epsilon,=) are decided in a weak fragment of ZF, and there is a 5 quantifier sentence that is not decided in ZFC (it is equivalent to a large cardinal axiom over ZFC). All of the axioms of ZF are an at most four quantifier sentence and an at most five quantifier axiom scheme. It has

been shown that AxC over ZF is equivalent to a five quantifier sentence (see Notre Dame Journal, not me). Show that over ZFC, any equivalence of the
continuum hypothesis requires a lot more quantifiers. Show that over ZFC, any statement consistent with ZFC that settles the continuum hypothesis, requires a lot more quantifiers.
...

Full text at http://cs.nyu.edu/pipermail/fom/2005-January/008707.html

David Corfield (2004)

... So much effort has been devoted to a thin notion of truth, so little to the thicker notion of significance. To say that scientists and mathematicians aim merely for the truth is a gross distortion. They aim for significant truths.

Full text - on *The Philosophy of Real Mathematics Page*.

Georg Cantor, August 28, 1899

...
[11] Cantor, by contrast, insists in his letter to Dedekind of August 28, 1899 that even finite multiplicities cannot be proved to be consistent. The fact of their consistency is a simple, unprovable truth - "the axiom of arithmetic"; the fact of the consistency of multiplicities that have an aleph as their cardinal number is in exactly the same way an axiom, "the axiom of the extended transfinite arithmetic".

Wilfried Sieg. Hilbert's programs: 1917-1922. *The Bulletin of Symbolic Logic*, March 1999, Vol.5, N 1 (online copy).

From: **Jeffrey Ketland** ...
Sent: Tuesday, August 31, 2004 3:30 AM
Subject: Re: [FOM] Proof "from the book"

... If I remember right, the gist is this. In studying the consistency problem, Gödel wanted initially to give an interpretation of second-order arithmetic within first-order arithmetic, and tried to find a definition of (second-order!) arithmetic truth in the first-order language. He discovered however that even first-order arithmetic truth is not arithmetically definable: i.e., what we now call Tarski's Indefinability Theorem. But, as he also discovered, the concept "provable-in-F", with F some fixed formal system, is arithmetically definable. This implies that arithmetic truth is distinct from provable-in-F, for any formal system F. This then gives us the quick proof of Gödel's first incompleteness theorem.
...

Full text at http://cs.nyu.edu/pipermail/fom/2004-August/008432.html

Benjamin Peirce & Son (1870+)

... in 1870 Benjamin Peirce defined mathematics as "the science that draws necessary conclusions" (see his son C.S.Peirce 1898/1955, p.137). C.S.Peirce himself described the work of a mathematician as composed of two different activities (p.138): (1) framing of a hypothesis stripped of all features which do not concern the drawing of consequences from it, without caring whether this hypothesis agrees with the actual facts; (2) drawing the necessary consequences from the hypothesis. He noted (Peirce 1902/1955, p.144) the difficulty to distinguish between two definitions of mathematics, one by its method ("drawing necessary conclusions"), another by its aim and subject matter ("the study of hypothetical state of things").

See p.5 of
Alexander Khait. The Definition of Mathematics: Philosophical and Pedagogical Aspects. *Science & Education* 00: 1-23, 2004, *Kluwer Academic Publishers*

Philip J. Davis and **Reuben Hersh** (1987):

In the real world of mathematics, a mathematical paper does two things. It testifies that the author has convinced himself and his friends that certain "results" are true, and presents a part of the evidence on which this conviction is based.
...
The Automath approach represents an unrealizable dream. ... the accepted practice of the mathematical community has hardly changed, except for the enlargement of the computer component.

... The myth of totally rigorous, totally formalized mathematics is indeed a myth.

Davis, P. J. & Hersh, R.. *Rhetoric and mathematics*. In J. S. Nelson, A. Mcgill & D. N. McCloskey (Eds.), *The rhetoric of the human sciences*. Madison: University of Wisconsin, 1987, pp. 53-69. (Thanks toWilliam J. Greenberg.)

[Added December 7, 2008. Thanks to Maris Ozols.] Now, 20 years later, the situation is changing... See *Notices of the AMS*, Special Issue on Formal Proof, Vol. 55, N 11, 2008 (available online).

From: **John McCarthy**...
Sent: Saturday, May 15, 2004 1:43 PM
Subject: Re: [FOM] Freeman Dyson on Inexhaustibility

Maybe physics is inexhaustible, but maybe it isn't. Here's why it might not be. Consider the Life World based on Conway's Life cellular automaton. It has been shown that self-reproducing universal computers are possible as configurations in the Life World. Therefore, one could have physicists in the Life World, but their physics would not be inexhaustible. They could discover or at least conjecture that their fundamental physics was a particular cellular automaton. However, their mathematics could be the same as ours - and therefore inexhaustible.

Full discussion thread - see **Foundations of Mathematics (FOM)** e-mail list.

The Continuum Hypothesis (I), 2000, by **W. Hugh Woodin**

"The current situation is the following.
We can build *models* of set theory with significant control over what is true in the model.
- During the 35 years since Cohen's work a great number of set theoretical propositions have been shown to be independent. Further problems in other areas of mathematics have also been shown to be independent.
This, as of yet, cannot be accomplished for models of number theory. The intuition of a *true* model of number theory remains unchallenged."

Another version:

"An important point is that neither Cohen's method of extension nor Godel's method of restriction affects the arithmetic statements true in the structures, so the intuition of a *true* model of number theory remains unchallenged.
It seems that most mathematicians do believe that arithmetic statements are either true or false. No generalization of Cohen's method has yet been discovered to challenge this view. But this is not to say that such a generalization will never be found."

See p. 568 of
W. Hugh Woodin. The Continuum Hypothesis, Part I, *Notices of the ACM*, Vol. 48, N6 (June/July 2001), pp. 567-576 (try online reading), Part II, Vol. 48, N7 (July 2001), pp. 681-690 (try online reading).

From: **Harvey Friedman** ...
Sent: *Tue Jan 20 01:17:00 EST 2004*
Subject: [FOM] On Foundational Thinking 1
...
(To avoid confusion, I draw a distinction between foundations of mathematics and mathematical logic. The latter consists of various mathematical spinoffs from foundations of mathematics, where one deemphasizes foundational thinking, and emphasizes mathematical adventures, including connections with various branches of mathematics.)
...

Full text at http://cs.nyu.edu/pipermail/fom/2004-January/007808.html

Ernest Gellner
Plough, Sword and Book: The Structure of Human History, University of Chicago Press, 1988, p.123

When knowledge is the slave of social considerations, it defines a special class; when it serves its own ends only, it no longer does so. There is of course a profound logic in this paradox: genuine knowledge is egalitarian in that it allows no privileged source, testers, messengers of Truth. It tolerates no privileged and circumscribed data. The autonomy of knowledge is a leveller.

Thanks to William J. Greenberg. Quoted after *Anthropological Wit and Wisdom*, by *Steve Froemming*.

Date: Fri, 26 Dec 2003 14:44:47 -0300
From: **Julio Gonzalez Cabillon** ...
Subject: [FOM] quotation from Weyl

Dear Roman Murawski,

I think that the passage you are seeking in Weyl's writings is:

"We now come to the decisive step of mathematical abstraction: we forget about what the symbols stand for. The mathematician is concerned with the catalogue alone; he is like the man in the catalogue room who does not care what books or pieces of an intuitively given manifold the symbols of his catalogue denote. He need not be idle; there are many operations which he may carry out with these symbols, without ever having to look at the things they stand for."

which is contained in the classic "The Mathematical Way of Thinking", an address given by Hermann Weyl at the Bicentennial Conference at the University of Pennsylvania, in 1940. It was first published in _Science_, in 1940, volume 92, pp. 437-446, and later reproduced in
James R. Newman's "The World of Mathematics" (volume 3).

Very best wishes to you all for 2004.

Julio

Context - at http://cs.nyu.edu/pipermail/fom/2003-December/007721.html

Doron Zeilberger, November 26, 2001

"... the conventional wisdom, fooled by our misleading "physical intuition", is that the real world is *continuous*, and that discrete models are necessary evils for approximating the "real" world, due to the innate discreteness of the digital computer.

Ironically, the opposite is true. The

REAL REAL WORLDS (Physical and MATHEMATICAL) ARE DISCRETE.

Continuous analysis and geometry are just degenerate approximations to the discrete world, made necessary by the very limited resources of the human intellect. While discrete analysis is conceptually simpler (and truer) than continuous analysis, technically it is (usyally) much more difficult."

"Real" Analysis is a Degenerate Case of Discrete Analysis by **Doron Zeilberger** (appeared in "New Progress in Difference Equations" (Proc. ICDEA 2001), Taylor and Francis, London, 2004)

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